# NAG Toolbox for MATLAB

# f07fe

## 1 Purpose

f07fe solves a real symmetric positive-definite system of linear equations with multiple right-hand sides,

$$AX = B,$$

where $A$ has been factorized by f07fd.

## 2 Syntax

```
[b, info] = f07fe(uplo, a, b, 'n', n, 'nrhs_p', nrhs_p)
```

## 3 Description

f07fe is used to solve a real symmetric positive-definite system of linear equations $AX = B$, this function must be preceded by a call to f07fd which computes the Cholesky factorization of $A$. The solution $X$ is computed by forward and backward substitution.

If **uplo** = 'U', $A = U^{\mathrm{T}}U$, where $U$ is upper triangular; the solution $X$ is computed by solving $U^{\mathrm{T}}Y = B$ and then $UX = Y$.

If **uplo** = 'L', $A = LL^{\mathrm{T}}$, where $L$ is lower triangular; the solution $X$ is computed by solving $LY = B$ and then $L^{\mathrm{T}}X = Y$.

## 4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates how $A$ has been factorized.

**uplo** = 'U'

$A = U^{\mathrm{T}}U$, where $U$ is upper triangular.

**uplo** = 'L'

$A = LL^{\mathrm{T}}$, where $L$ is lower triangular.

*Constraint*: **uplo** = 'U' or 'L'.

2: **a**(**lda**,$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The Cholesky factor of $A$, as returned by f07fd.

3: **b**(**ldb**,$*$) **– double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The $n$ by $r$ right-hand side matrix $B$.

### 5.2 Optional Input Parameters

1: **n – int32 scalar**

*Default*: The second dimension of the array **a**.

$n$, the order of the matrix $A$.

*Constraint*: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

*Default*: The second dimension of the array **b**.

$r$, the number of right-hand sides.

*Constraint*: $\mathbf{nrhs\_p} \geq 0$.

### 5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb

### 5.4 Output Parameters

1: **b(ldb,$*$) – double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The $n$ by $r$ solution matrix $X$.

2: **info – int32 scalar**

**info** $= 0$ unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**info** $= -i$

If **info** $= -i$, parameter $i$ had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **nrhs_p**, 4: **a**, 5: **lda**, 6: **b**, 7: **ldb**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

## 7 Accuracy

For each right-hand side vector $b$, the computed solution $x$ is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if **uplo** $=$ 'U', $|E| \leq c(n)\epsilon \left|U^{\mathrm{T}}\right||U|$;
if **uplo** $=$ 'L', $|E| \leq c(n)\epsilon |L|\left|L^{\mathrm{T}}\right|$,

$c(n)$ is a modest linear function of $n$, and $\epsilon$ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution $x$ satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \left\||A^{-1}||A||x|\right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\||A^{-1}||A|\right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling f07fh, and an estimate for $\kappa_\infty(A)$ ($= \kappa_1(A)$) can be obtained by calling f07fg.

## 8 Further Comments

The total number of floating-point operations is approximately $2n^2r$.

This function may be followed by a call to f07fh to refine the solution and return an error estimate.

The complex analogue of this function is f07fs.

## 9 Example

```
uplo = 'L';
a = [4.16, 0, 0, 0;
     -3.12, 5.03, 0, 0;
     0.56, -0.83, 0.76, 0;
     -0.1, 1.18, 0.34, 1.18];
b = [8.699999999999999, 8.300000000000001;
     -13.35, 2.13;
     1.89, 1.61;
     -4.14, 5];
[a, info] = f07fd(uplo, a);
[bOut, info] = f07fe(uplo, a, b)
```

```
bOut =
    1.0000    4.0000
   -1.0000    3.0000
    2.0000    2.0000
   -3.0000    1.0000
info =
           0
```